INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS, VOL. 25, 125–142 (1996)

UNSTEADY DETACHED SEPARATION FROM A CIRCULAR CYLINDER PERFORMING ROTATIONAL OSCILLATIONS IN A UNIFORM VISCOUS INCOMPRESSIBLE FLOW

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SUMMARY

The origination of detached separation is studied on the basis of a numerical solution of the full Navier–Stokes equations. Fluxes of vorticity with different signs generated with twice the frequency of cylinder oscillation move from the cylinder to the outer surface of a detached liquid layer in the form of concentric rings. Near the critical layer between the attached layer and the main flow these rings are torn and crimped to the regions of separated vortices of the corresponding sign. The form of detached separated vortices is similar to that of vortices originating from a stationary circular cylinder in a uniform flow. Transition of the flow to a non-symmetric form with Karman vortex street generation at a Reynolds number (based on the radius) greater than 17 is revealed. This critical Reynolds number is smaller than that for a stationary circular cylinder in a viscous stream (where $Re = 20$ has been determined to be a critical value) and corresponds to the Reynolds number extrapolated from the critical value for the stationary cylinder by increasing the cylinder radius by the attached layer thickness. The vorticity flux from the cylinder surface immediately into the separation region decreases as the frequency of cylinder oscillation increases. Violation of the flow potentiality in the detached separation region is the main cause of the vorticity generation on the outer surface of the attached liquid layer. © 1997 by John Wiley & Sons, Ltd.

Int. J. Numer. Meth. Fluids, **25**, 125–142 (1997)

No. of Figures: 8. No. of Tables: 0. No. of References: 36.

KEY WORDS: viscous; vorticity; detached separation; cylinder

1. INTRODUCTION

Numerous types of flow separation are known. The basic feature that we are accustomed to regarding as a necessary part of flow separation is the presence of a solid surface where the separation has its origin. The phenomenon of detached separation differs from the conventional idea. This novel type of flow separation has been revealed in a natural experiment with a slotted wing and also obtained in a computational simulation of the viscous flow around a circular cylinder performing rotational oscillations in a uniform stream.[1–3] In both cases detached separation can be characterized, in the first place, by properties of the main flow when, depending on the body geometry and flow parameters, the global pressure field causes flow separation, and in the second place by the existence of a liquid layer or jet which separates the recirculation region from the body surface. In the case of a circular cylinder performing rotational oscillations in a uniform stream, different types of detached separation were

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CCC 0271–2091/97/020125–18 $17.50

Received March 1995
Revised February 1996

obtained: symmetrical detached separation at low Reynolds numbers[2,3] and development of unsteady detached separation with generation of a vortex street similar to the Karman vortex street.[4] If the Reynolds number Re (based on the radius of the cylinder) exceeds 200, the attached liquid layer separating the recirculation region becomes thinner and alternation of conventional flow separation with detached flow separation begins at certain values of the cylinder oscillation frequency and amplitude. At the same Reynolds number the outer part of the attached layer can destruct to generate small-scale vortices which convect with the flow into the detached separation region and add to large-scale vortices typical of the Karman vortex street.[4,5] These detached separation forms are essential physical phenomena requiring a more detailed description than References 1–5 provide.

Interesting detached separation detection by computer simulations is due to Taneda's experiment,[6] where the viscous flow around a rotationally oscillating circular cylinder was studied. In this natural experiment for $Re_D = 35$ (Reynolds number based on the cylinder diameter) the method of electrochemical emission of a tracing substance from the cylinder surface into the flow was used and an intriguing flow pattern (Figure 1) was obtained.[6] The tracing particles enter the boundary layer generated around the oscillating cylinder, move in the layer, are displaced to the rear critical point of the flow around the cylinder and form a thin layer of the tracing substance that moves along the flow symmetry axis. It should be emphasized that even at $t = 12D/U_\infty$ (where D is the cylinder diameter and U_∞ is the velocity of the undisturbed flow) this layer remains as a straight line, which demonstrates that the flow behind the cylinder is symmetrical. Consequently, it has been conjectured in Reference 6 that the cylinder oscillations stabilize the flow and prevent separation on the cylinder surface. A different assumption has since been proposed in Reference 2: separation near the cylinder does exist and develops from the outer boundary of a liquid layer attached to the cylinder, which is clearly seen in Figure 1. This assumption is based on the idea that the development of a boundary layer changes insignificantly the geometry of the effective body (the cylinder plus the attached layer).[7] As a consequence, the unfavourable pressure gradient of the main flow must initiate flow separation on the outer surface of the attached liquid layer. In view of such physical processes the

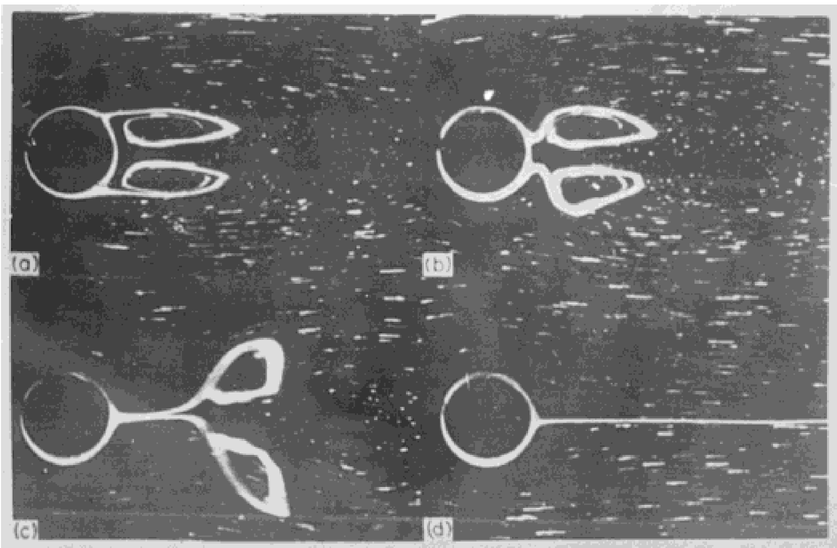

Figure 1. Streamlines and streaklines around a circular cylinder performing a rotatory oscillation about its axis in a uniform flow: $D = 1$ cm, $U = 0{\cdot}33$ cm s^{-1}, $Re_D = 35$, $N = 2$ Hz, $ND/U = 6$, $A = \pi/4$, $x = Ut$, $x/d =$ (a) 0, (b) 1·5, (c) 3·4, (d) 12 (from Reference 6)

flow topology in Figure 1 may be considered as the first indication of the existence of an attached layer. However, some novel physical phenomena must be discovered and ideas proposed in order to understand whether it is possible to prevent flow separation from the effective body.

Attempts are being made to develop theoretical models describing the phenomenon presented in Figure 1. In the framework of the boundary layer approach it is shown by Ruban and Kravtsova[8] that the rotational oscillation promote separation on the cylinder. Using the generalized Lagrangian mean theory, Wu *et al.*[9] have shown that the cylinder oscillations always reduce the vorticity flux from the cylinder in comparison with the corresponding case of steady flow. Moreover, when the period of cylinder oscillation is much less than that of flow separation, a critical amplitude of oscillation leads to zero mean flux of vorticity. The minimum value of the Strouhal number (hereafter referred to as reduced frequency K) based on the cylinder radius and the oscillation period for prevention of flow separation is 10. As opposed to this theoretical study, computational investigation[2] indicates a reduced frequency of three. The results of the computational experiment detailed in Reference 2 and in Section 3 of the present paper allow us to see the topology of equal-vorticity lines (Figure 2(a)) and streamlines (Figure 2(b)), which clearly show that the viscous flow computed for $K = 3$, $Re = 10$ and an effective amplitude $A = \pi/4$ includes an attached layer and a recirculation region. Of no less importance is the question of how the vorticity appears in the inner region of the flow, i.e. what is the physical process that leads to vorticity generation on the liquid surface—the outer boundary of the attached liquid layer? Discussion of this problem and assessment of the assumptions about the physics of the flow demand additional information to that in Reference 2 and Figure 2. The study of new, vast computational simulation results and the analysis of vorticity generation on the liquid boundary are the goals of the present investigation.

The paper hereafter consists of five sections. The problem statement for the viscous flow around a circular cylinder performing rotational oscillations in a uniform flow is presented in Section 2. The problem is formulated in terms of streamfunction and vorticity. Section 3 is a concise description of a finite difference computational algorithm for solving the above computational fluid dynamics problem. In Section 4 we consider the process of origination of detached separation and the conditions of development of non-symmetric flow. Simultaneously, theoretical premises for vorticity generation on the liquid boundary that separates the recirculation region and the attached liquid layer are analysed. The flow at low Reynolds numbers ($Re < 40$) is discussed in Section 5. The dependence of the flow type on the frequency of oscillation is studied. The attached liquid layer around the cylinder is determined to be a two-layer structure by studying trajectories of liquid particles in the flow. Such a structure is explained through physical effects similar to those underlying the design of a centrifugal pump. Finally, in section 6 we analyse peculiarities of the solution that lead to the formulation of new problems in the study of detached separation from bodies in viscous flows.

2. *Problem statement*

The basis of the study is the Navier–Stokes equations written in terms of the streamfunction ψ and vorticity Ω, defined through the velocity components u and v, namely the vorticity transport equation

$$\frac{\partial\Omega}{\partial t} + u\frac{\partial\Omega}{\partial x} + v\frac{\partial\Omega}{\partial y} = \frac{1}{Re}\Delta\Omega, \quad (1)$$

where Δ is the Laplace operator, $Re = U_\infty R/\nu$ is the Reynolds number and ν is the kinematic viscosity coefficient, and the Poisson equation for the streamfunction,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \Omega. \quad (2)$$

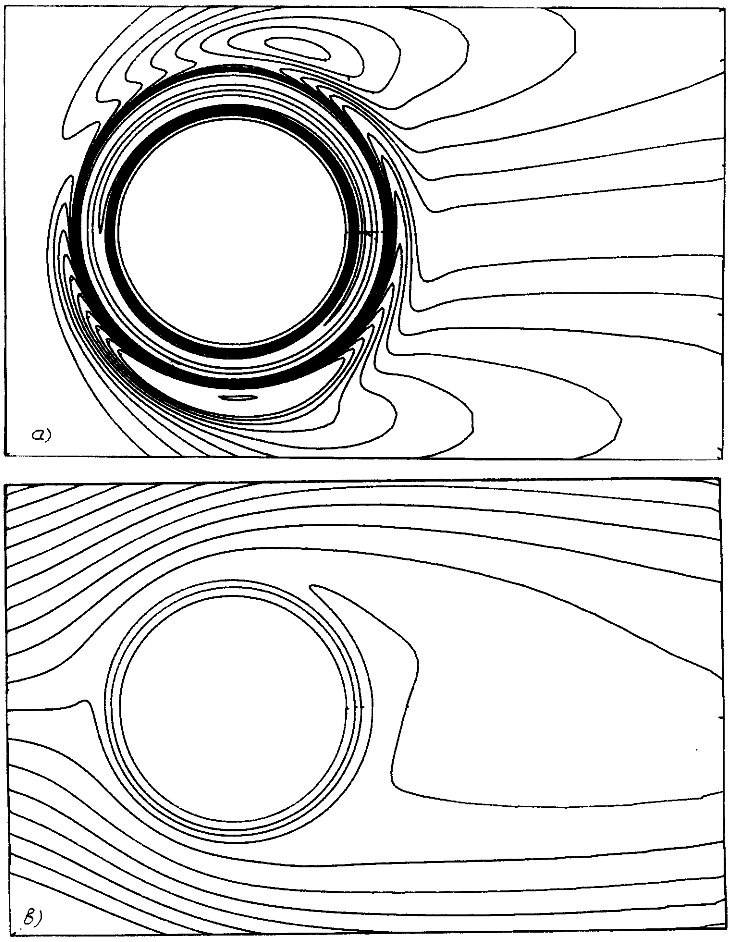

Figure 2. (a) Equal-vorticity lines and (b) streamlines around a circular cylinder for $Re = 10$, $K = 3$, $A = \pi/4$ at $t = 20$

The variables and parameters of the problem are non-dimensional with respect to R and U_∞.

For the problem of the viscous flow around a circular cylinder it is convenient to use polar co-ordinates (r, ϕ), where r is the radius and ϕ is the polar angle. The computational grid should be additionally compressed towards the cylinder surface by the transformation

$$r = e^{\xi}, \qquad \eta = \phi, \qquad H^2 = e^{2\xi}, \qquad \xi = d\tan(\pi\theta/2), \tag{3}$$

where $H^2 = (\partial x/\partial\xi)^2 + (\partial y/\partial\xi)^2$ is the Jacobian of transformation from Cartesian co-ordinates (x, y) to curvilinear orthogonal co-ordinates (ξ, η).

Over the cylinder surface s, no-slip boundary conditions are imposed:

$$v_r = 0, \qquad v_\phi = W(t), \tag{4}$$

where $v_r = -(1/r)\partial\psi/\partial\phi$ is the radial velocity component, $v_\phi = \partial\psi/\partial r$ is the circumferential velocity component and W is the cylinder rotation angular speed. The conditions (4) are reformulated in terms of the streamfunction as

$$\psi = 0, \qquad \frac{\partial\psi}{\partial r} = W(t). \tag{5}$$

The flow is assumed to be uniform at infinity (the conventional boundary condition). However, the computations in this research cover a finite region which is defined by the cylinder surface s and the outer boundary s_∞ (the circumference of radius R_∞). On the outer boundary the 'soft' boundary condition for vorticity is used, i.e.

$$\frac{\partial \Omega}{\partial \xi} = 0, \tag{6}$$

and the tangential velocity is prescribed by

$$H^{-1} \frac{\partial \psi}{\partial \xi} = U \sin \eta. \tag{7}$$

The boundary condition (6) is a standard one and has not raised doubts yet in complex cases of large symmetrical lengthwise separation behind stationary circular cylinders.[10] Equation (7) is a conventional condition for the study of the initial phase of the viscous flow around a cylinder[11] and is preferable to prescribing a streamfunction on s_∞[12] when the long-time flow development is studied. Additional terms such as a sink[10] are included in the study of the steady flow and finally a completely 'soft' boundary condition is derived in the study of the unsteady flow past an elliptic cylinder or an aerofoil.[13,14] An influence of the boundary condition (7) on the frequency of vortex shedding from the cylinder is possible and may be essential; the same is true for the drag and lift of the cylinder. However, for the study of the initial phase of detached separation this boundary condition may be used with confidence, and for the study of the general character of unsteady detached separation we can again use the condition (7) with the intention of elaborating the 'open-ended' condition with additional terms for more complex investigations.[15]

Along with the boundary conditions we should discuss the following problem: a solution to the modified N–S equations (1) and (2) written in variables ψ and Ω is not always a solution to the initial set of equations in variables u, v and p; this was clearly shown by Zakharenkov.[16]

This distinction is usually established easily when the pressure field p is calculated from the N–S equations in the Gromeka–Lamb form

$$\begin{aligned} \frac{\partial p}{\partial \eta} &= -\frac{1}{Re}\frac{\partial \Omega}{\partial \xi} + \frac{\partial}{\partial \xi}\left(\frac{\partial \psi}{\partial t}\right) - \frac{\partial}{\partial \eta}\left(\frac{V^2}{2}\right) + \Omega \frac{\partial \psi}{\partial \eta}, \\ \frac{\partial p}{\partial \xi} &= \frac{1}{Re}\frac{\partial \Omega}{\partial \eta} - \frac{\partial}{\partial \eta}\left(\frac{\partial \psi}{\partial t}\right) - \frac{\partial}{\partial \xi}\left(\frac{V^2}{2}\right) + \Omega \frac{\partial \psi}{\partial \xi}, \end{aligned} \tag{8}$$

where the streamfunction ψ and vorticity Ω are known from equations (1) and (2). Therefore the requirement that the solution to the derived system be equivalent to the initial N–S system solution is formulated as a requirement for the pressure p not to depend on the path of integration of equations (8).[16] In particular, the pressure distribution on the body surface is analysed in References 3 and 17.

For a complex problem such as the viscous flow around a circular cylinder performing rotational oscillations, the requirement for the pressure field not to depend on the path of integration of equations (8) is very difficult. In fact, the steady viscous flow around a circular cylinder rotating at a constant angular velocity in a viscous fluid which is at rest at infinity is described in terms of the velocity potential of a single vortex. However, the unsteady viscous flow around a cylinder rotating in an initially resting liquid is described by a complex relation including Bessel functions.[18] To strictly solve this problem in the frame of the present problem statement, one should specify a time-dependent low tangential velocity over s_∞[18]. A very small variation in $v_\phi(R_\infty, \eta)$ can produce a large discrepancy in the pressure on the cylinder surface.

When a circular cylinder is rotated at a constant angular velocity in a uniform flow, a lateral Magnus force is generated. In a plane flow with a uniform velocity directed along the x-axis, this force is F_y directed along the y-axis. At high time-dependent angular velocities this force is large and changes its sign depending on the angular speed. Such behaviour of the integral characteristic is associated with a complex time-dependent pressure field near the cylinder; it is, however, obvious that this pressure field is not associated with notable variations in the velocity v_ϕ at the outer boundary s_∞ of the calculation domain. Nevertheless, it is possible to construct an interval assessment of the solution quality by proceeding in the following way. In the first case we specify the correction function v_ϕ on s_∞,[16] while in the other case we use the surface vorticity correction that provides the correct pressure calculations.[17] This interval estimation was developed by Zakharenkov[2] and good coincidence of the flow topology (i.e. patterns of both equal-vorticity lines and streamlines) has been obtained, a slight difference appearing only in a very small region near the cylinder in higher derivatives of the streamfunction and vorticity. It is obvious that, by analogy with modelling the curvilinear tangential discontinuity in an aerofoil wake (where the centrifugal force is balanced by the cross-gradient of pressure and the solution contains distributed sinks[19]), the same correcting field of distributed sinks may be used in our problem for modelling the large cross-force F_y appearing in the thin distorted boundary layer. Nevertheless, in the initial phase of investigation it is useful to employ the approach derived by Zakharenkov[2] and first estimate the qualitative properties of the phenomenon under consideration. Note that for the flow in an aerofoil wake the above-mentioned corrections are like the second-order approach and this may be expected to appear in our problem.

Returning to the formulation of the problem of the viscous flow around a circular cylinder performing rotational oscillations, let us specify the angular speed by the law[2]

$$W = \tfrac{1}{2}Aw\sin[w(t - t_0)], \qquad w = 2\pi K, \qquad K = R/U_\infty T, \tag{9}$$

where K is the reduced frequency, A is the amplitude of oscillation and T is the period of oscillation. For example, the value $K = 3$ corresponds to a flow in which three full oscillations of the cylinder are accomplished during the time interval when free particles in the far field travel a distance equal to the cylinder radius.

The system is initially at rest. In going to the velocity U_∞, the viscous flow accelerates from rest either instantaneously or by following a parabolic law[11,20] for time $T_0 = 0{\cdot}2$:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \begin{cases} -0, & t < 0, \\ -4t/T_0^2, & 0 \leqslant t < T_0/2, \\ 4(t - T_0)/T_0^2, & T_0/2 < t < T_0, \\ 0, & T_0 < t. \end{cases}$$

Within this problem statement the parameter of Taneda's experiment are Reynolds number $Re = 17{\cdot}5$ ($Re_D = 35$), amplitude $A = \pi/4$ and reduced frequency $K = 3$. Finally, the study by Wu *et al.*[9] suggests that flow separation will be eliminated if $K \gg 10$.

3. NUMERICAL SOLUTION METHOD

The above problem is very complex for computational simulation; this is evidenced by the availability of only a few examples of solutions in the literature. Nevertheless, the results of numerical simulations will be more persuasive if a reliable conventional computational algorithm is used. Such is the algorithm developed in References 2 and 20. The basic steps of the computational procedure are as follows.

Equations (1) and (2) of the N–S system are decoupled and solved independently of one another. The Poisson equation (2) is solved by the direct method (i) after expansion into a Fourier series (a trigonometrical polynomial) in the cyclic co-ordinate η and (ii) using the fast Fourier transform.[21] This method provides the exact solution of the system of algebraic equations for finding the streamfunction on a grid. This system is composed of three-point linear equations written for Fourier components of the grid streamfunction:

$$A\hat{\psi}_{j-1} - 2B\hat{\psi}_j + C\hat{\psi}_{j+1} = F_j, \quad j = 1, 2, \ldots, N_\xi. \tag{10}$$

Here the coefficients depend on r.

The vorticity transport equation (1) is solved by the alternating direction method:[22,23]

$$\begin{aligned} H^2\frac{\tilde{\Omega}-\Omega^n}{\tau/2} &= \frac{1}{Re}\frac{\partial^2\tilde{\Omega}}{\partial\eta^2} + \left(\frac{\partial\psi}{\partial\xi}\right)^{n+1/2}\frac{\partial\tilde{\Omega}}{\partial\eta} + \frac{1}{Re}\frac{\partial^2\Omega^n}{\partial\xi^2} - \left(\frac{\partial\psi}{\partial\eta}\right)^n\frac{\partial\Omega^n}{\partial\xi},\\ H^2\frac{\Omega^{n+1}-\tilde{\Omega}^n}{\tau/2} &= \frac{1}{Re}\frac{\partial^2\tilde{\Omega}}{\partial\eta^2} + \left(\frac{\partial\psi}{\partial\xi}\right)^{n+1/2}\frac{\partial\tilde{\Omega}}{\partial\eta} + \frac{1}{Re}\frac{\partial^2\Omega^{n+1}}{\partial\xi^2} - \left(\frac{\partial\psi}{\partial\eta}\right)^{n+1}\frac{\partial\Omega^{n=1}}{\partial\xi}, \end{aligned} \tag{11}$$

where n is the time step index, τ is the time step length and $A^{n+1/2} = (A^n + A^{n+1})/2$.

The second derivatives in (2) and (11) are approximated by central differences. The velocities v_ξ and v_η in (11) are calculated on the basis of the streamfunction, also by using central differences. The non-linear terms in (11) are approximated by three-point upwind differences,[24] e.g.

$$\frac{\partial}{\partial x}\left[\left(\frac{\partial\psi}{\partial y}\right)\Omega\right]_j = \frac{1}{2h_x}\left\{-3\left(\frac{\partial\psi}{\partial y}\right)_j\Omega_j + \left[3\left(\frac{\partial\psi}{\partial y}\right)_{j+1} + \left(\frac{\partial\psi}{\partial y}\right)_j\right]\Omega_{j+1} - \left(\frac{\partial\psi}{\partial y}\right)_{j+1}\Omega_{j+2}\right\}. \tag{12}$$

The peculiarity of the solution of the decoupled equations (1) and (2) is the necessity to reformulate the no-slip boundary condition (4) in a form which contains a suitable boundary condition for the vorticity transport equation.[23] The theory of the computational realization of the no-slip condition as a boundary condition for the vorticity and the Dirichlet boundary condition for the streamfunction is derived in References 20, 25 and 26. The investigation by Tarunin[25] shows the existence of an optimal approximation formula for the vorticity on a solid boundary. The uniformity of all approaches derived earlier for the determination of the vorticity on a solid boundary has been shown by Zakharenkov.[26] Thereafter the theory has been developed for other forms of the N–S equations[16] (in velocity–vorticity variables and velocity–pressure variables, when the Poisson equation is solved for pressure). The universal two-parameter approximation formula for the vorticity on the boundary s takes the form

$$\begin{aligned} &\left.\frac{\partial\psi}{\partial y}\right|_s = L\psi - E\left.\frac{\partial^2\psi}{\partial y}\right|_s + \frac{h_y(2h_y-\alpha)}{3!}\left.\frac{\partial^3\psi}{\partial y^3}\right|_s + \frac{h_y^2(6h_y-\beta)}{4!}\left.\frac{\partial^4\psi}{\partial y^4}\right|_s + \cdots,\\ &L\psi = h_y^{-2}\sum_{j=0}^{3}K_j\,\psi_j, \qquad E = (6\alpha-\beta)/22,\\ &K_0 = (-198h_y + 50\alpha - 12\beta)/132, \qquad K_1 = (264h_y - 114\alpha + 30\beta)/132,\\ &K_2 = (-66h_y + 78\alpha - 24\beta)/132, \qquad K_3 = (-14\alpha + 6\beta)/132,\\ &\left.\left(\frac{\partial^2\psi}{\partial y^2}\right)\right|_s = \Omega|_s, \qquad \left.\left(\frac{\partial^3\psi}{\partial y^3}\right)\right|_s = \left.\frac{\partial\Omega}{\partial y}\right|_s = \frac{-3\Omega_0 + 4\Omega_1 - \Omega_2}{2h_y}, \end{aligned} \tag{13}$$

where α and β are parameters.